GCE

Further Mathematics A

Y540/01: Pure Core 1

A Level

Mark Scheme for June 2023

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

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MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. **Annotations**

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

5. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

   For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

   It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   Award NR (No Response)
   - if there is nothing written at all in the answer space and no attempt elsewhere in the script
   - OR if there is a comment which does not in any way relate to the question (e.g. ‘can’t do’, ‘don’t know’)
   - OR if there is a mark (e.g. a dash, a question mark, a picture) which isn’t an attempt at the question.

   Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

   If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

   If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

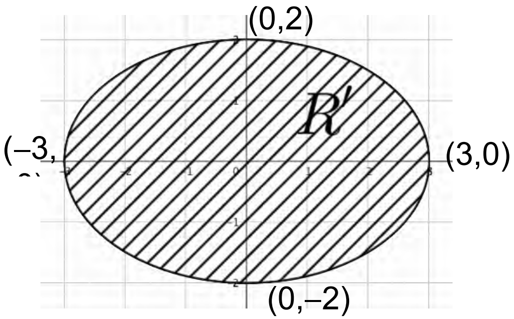
j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1			**DR**		
			$\sum_{1}^{50} r^2(16-r)=\sum_{1}^{50}16r^2-\sum_{1}^{50}r^3$	M1	Separating and using correct formulae for $\sum r^3$ and $\sum r^2$
			$=\frac{16}{6}\times 50\times 51\times 101-\frac{1}{4}50^2\times 51^2$	M1	Substituting must be seen
			$=(686800-1625625)$		
			$=-938825$	A1	Including all notation correct (Sigmas do not need limits)
			Alternative method:		
			$\sum_{1}^{n} r^2(16-r)=\sum_{1}^{50}16r^2-\sum_{1}^{50}r^3$	M1	Separating and using the correct formulae
			$=\frac{16}{6}n(n+1)(2n+1)-\frac{1}{4}n^2(n+1)^2$		
			$=n(n+1)\left(\frac{8}{3}(2n+1)-\frac{1}{4}n(n+1)\right)$		
			$=\frac{n(n+1)}{12}\left(32+61n-3n^2\right)$		
			$=\frac{50\times 51}{12}\left(32+61\times 50-3\times 50^2\right)$	M1	Substituting anywhere in the algebra
			$=-938825$	A1	
				[3]	

Question			Answer	Marks	Guidance
2	(a)		**DR** $(w-1)^4+4(w-1)^3+......=0$ $\Rightarrow w^4-4w^3+6w^2-4w+1+.......$ $\Rightarrow w^4+3w^2+2=0$	**M1** **M1** **A1**	Substitute $z = w - 1$ Expanding with at least $(w–1)^4$ seen Convincingly shown **AG**. Must include = 0 on last line. Brackets must be fully expanded in working or evidence of collection of like terms A0 if variable used is not w.
			Alternative method: $(z+1)^4+3(z+1)^2+......=0$ $\Rightarrow z^4+4z^3+.......$ $\Rightarrow z^4+4z^3+9z^2+10z+6=0=0$	**M1** **A1**	Substitute $w = z + 1$ into end result
			Alternative method using symmetry of roots $\sum\alpha=-4,\ \sum\alpha\beta=9.\ \sum\alpha\beta\gamma=-10,\ \alpha\beta\gamma\delta=6$ $\sum(\alpha+1)=\sum\alpha+4=-4+4=0$ $\sum(\alpha+1)(\beta+1)=\sum\alpha\beta+3\sum\alpha+6=9-12+6=3$ For $\sum(\alpha+1)(\beta+1)(\gamma+1)=0$ and $(\alpha+1)(\beta+1)(\gamma+1)(\delta+1)=2$	**B1** **B1** **B1**	For sums from original equation and finding the sum of the new roots For showing convincingly the sum of new roots in pairs For the last two
				[3]	
	(b)		$w^2=-1,-2$ $\Rightarrow w=\pm\mathrm{i},\pm\sqrt{2}\mathrm{i}$ $\Rightarrow z=\pm\mathrm{i}-1,\pm\sqrt{2}\mathrm{i}-1$	**M1** **M1** **A1**	Solving quadratic equation in w^2 (or using their variable) Square rooting their w^2, including $\pm$, as long as their w^2 not both non-negative and real. cao Answers with no working is 0
				[3]	

Question			Answer	Marks	Guidance
3	(a)		$r=\frac{1}{2}\sqrt{(-3)^2+(\sqrt{3})^2}=\frac{1}{2}\sqrt{12}=\sqrt{3}$	B1	AG so must show use of $\lvert z\rvert=\sqrt{a^2+b^2}$
			$\arctan\left(-\frac{\sqrt{3}}{3}\right)=-\frac{\pi}{6}$ $\Rightarrow \theta=-\frac{\pi}{6}+\pi=\frac{5\pi}{6}$	B1	**AG**. Or $\theta=\pi-\arctan\left(\frac{\sqrt{3}}{3}\right)$, may be indicated on a diagram, but clear reasoning must be shown (eg.finding complementary angle, or use of Pythagoras' theorem and then arcsin or arccos)
			Alternative method: $\sqrt{3}e^{\frac{5}{6}\pi i}=\sqrt{3}\left(\cos\left(\frac{5}{6}\pi\right)+i\sin\left(\frac{5}{6}\pi\right)\right)$	M1	
			$=\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=\frac{\sqrt{3}i-3}{2}$	A1	**AG** Clearly shown
				[2]	
	(b)		$r=\left(9\sqrt{3}\right)^{\frac{1}{5}}=\left(3^{\frac{5}{2}}\right)^{\frac{1}{5}}=\sqrt{3}$	B1	For $r=\sqrt{3}$ **oe** (including 1.73....)
			$\theta=\frac{1}{5}\left(\frac{5}{6}\pi+2r\pi\right)=\frac{\pi}{30}(5+12r)$ for $r=0,1,2,3,4$	M1	For their $\frac{5}{6}\pi+2\pi n$ from (a) divided by 5 (either in terms of n, or for at least two values of n).
			$\Rightarrow z=\sqrt{3}e^{\frac{1}{6}\pi i}, \sqrt{3}e^{\frac{17}{30}\pi i}, \sqrt{3}e^{\frac{29}{30}\pi i}, \sqrt{3}e^{\frac{41}{30}\pi i}, \sqrt{3}e^{\frac{53}{30}\pi i}$	A1	Allow $\sqrt{3}e^{\frac{1}{30}(5+12n)\pi i}$ for n = 0, 1, 2, 3, 4. Accept only $r=\sqrt{3}$ or $(3)^{\frac{1}{2}}$ For last two marks, If M0 then SC B1 for all five roots
				[3]	

Question			Answer	Marks	Guidance
4	(a)		$\mathbf{BA} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ i.e. a reflection in the x-axis	M1 A1	Multiplication in correct order Or reflection in the line $y = 0$ Answer with no working B2
			Alternative method: A represents a rotation anti-clockwise of 90^0 B represents a reflection in the line $y = x$ Taken one after the other gives a reflection in the x- axis	M1 A1	
				[2]	
	(b)		T_A is a rotation 90 degrees (anti-clockwise about O) (423 has remainder 3 when divided by 4) so $\mathbf{A}^{423} = \mathbf{A}^3$ So $\mathbf{A}^{423} = \mathbf{A}^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	B1 M1 A1	 For $\mathbf{A}^4 = \mathbf{I}$ or T_A repeated four times is a 360 degree rotation. Condone clockwise instead of anticlockwise for T_A so notes that 423 is divisible by 4 with remainder 3 so $\mathbf{A}^{423} = \mathbf{A}^3$ As $\mathbf{A}^3$ represents a 270 degrees rotation anti-clockwise (or 90 degrees clockwise) or by direct calculation of $\mathbf{A}^3$.
				[3]	
	(c)	(i)	$\text{Det}\mathbf{C} = \frac{1}{6}$ $\Rightarrow$ Area of R' $= \frac{1}{6} \times 36\pi = 6\pi$	M1 A1	$\frac{1}{3} \times \frac{1}{2}$ or $\frac{1}{6}$ seen. Or area of ellipse $= \pi ab = \pi \times 2 \times 3$ cao
				[2]	

Question			Answer	Marks	Guidance
4	(c)	(ii)		**M1**	Correct shape, (approximately elliptical, possibly identified by scales, closed, radius on horizontal axis > radius on vertical axis)
				A1	Intercepts labelled at $x = \pm 3$
				A1	and $y = \pm 2$(Give A1only rather than A2 if only positive intercepts are labelled on both) Give A1 if x and y axes interchanged.
				A1	Correctly shaded or labelled R' and everything else correct.
				[4]	

Question			Answer	Marks	Guidance
5	(a)		Auxiliary equation: $n^2-2n+5=0$ $\Rightarrow n=1\pm 2\text{i}$ $\Rightarrow y=\text{e}^x\left(A\cos 2x+B\sin 2x\right)$ **oe**	B1 B1	Correct roots of auxiliary equation. **ft** complex k only. Or $y=R\text{e}^x\cos(2x+\varphi)$ or $y=R\text{e}^x\sin(2x+\varphi)$ Or $y=A\text{e}^{(2\text{i}+1)x}+B\text{e}^{(-2\text{i}+1)x}$ Final equation must be $y=\text{f}(x)$
				[2]	
	(b)		Trial function: $y=ax^2+bx+c$ $\Rightarrow y'=2ax+b,\quad y''=2a$ $\Rightarrow 2a-2(2ax+b)+5(ax^2+bx+c)\equiv 4x-5x^2$ $\Rightarrow 5ax^2+x(-4a+5b)+5c-2b+2a\equiv 4x-5x^2$ $\Rightarrow a=-1, b=0, c=\frac{2}{5}$ $\Rightarrow$ GS: $y=\text{e}^x\left(A\cos 2x+B\sin 2x\right)-x^2+\frac{2}{5}$	B1 M1 A1 A1ft	Allow any extraneous terms in the trial function (eg.dx^3) as long as d shown to be zero, $a,b,c\neq 0$ Differentiates their trial function to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and substitutes **ft** *their* particular integral, and their CF from (a) (dependent on CF containing exactly two arbitrary constants)
				[4]	

Question			Answer	Marks	Guidance
6			**DR** Mean value $= \dfrac{1}{0.5}\displaystyle\int_0^{0.5}\left(20-20\tanh(1.44t)\right)\mathrm{d}t$	**B1**	For using the definition of the mean value of p wrt t, correct limits and 1/0.5.
			$= \left[40t - \dfrac{40}{1.44}\ln\left(\left\|e^{1.44t}+e^{-1.44t}\right\|\right)\right]_0^{0.5}$	**M1**	For $\int \tanh 1.44t \, \mathrm{d}t = k\ln\|e^{1.44t}+e^{-1.44t}\|(+c)$ k can = 1 Or $\int \tanh 1.44t \, \mathrm{d}t = k\ln\|\cosh 1.44t\|(+c)$ May see integration by substitution, eg. $u = e^{1.44t}+e^{-1.44t}$ or $u = \cosh 1.44t$. If so, award this mark for $k\ln\|u\|$ seen
			$= \left(20 - \dfrac{40}{1.44}\left(\ln\left(e^{0.72}+e^{-0.72}\right)-\ln 2\right)\right)$	**A1**	For fully correct integration of $\tanh 1.44t$. Either for $\int \tanh 1.44t \, \mathrm{d}t = \frac{1}{1.44}\ln\|e^{1.44t}+e^{-1.44t}\|(+c)$ or $\int \tanh 1.44t \, \mathrm{d}t = \frac{1}{1.44}\ln\|\cosh 1.44t\|(+c)$ or $\int \tanh 1.44t \, \mathrm{d}t = \frac{1}{1.44}\ln\|u\|\,(+c)$ with u as above. Condone missing modulus.
			$= \left(20 - \dfrac{40}{1.44}\ln\dfrac{2.5412}{2}\right) = 13.35\text{(W)}$	**A1**	cao, with clear working.
				[4]	

Question			Answer	Marks	Guidance
7	(a)	(i)	e.g. sign of $\frac{\mathrm{d}y}{\mathrm{d}t}$ is +ve because the force is in the same direction of motion	B1	Convincingly shown
				[1]	
		(ii)	$\frac{1}{2}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 5\cosh 0 - 3 - 5$	M1	Substitutes $t = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = \pm 5$
			$= -3$ $\Rightarrow \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -6(\mathrm{m}s^{-2})$	A1	Or 6 towards O
				[2]	
	(b)	(i)	Maclaurin: $x = \mathrm{f}(0) + \mathrm{f}'(0)t + \ldots$ When $t = 0, (x = 6 =)\,\mathrm{f}(0) = 6, \left(\frac{\mathrm{d}x}{\mathrm{d}t} = \right) = \mathrm{f}'(0) = -5$ $\Rightarrow x = 6 - 5t\ldots.$	B1	First two terms ofMaclaurin's formula, possibly in generalised form, must be seen **AG** Convincingly shown.
				[1]	
		(ii)	3rd term of Maclaurin is $\mathrm{f}''(0)\frac{t^2}{2!}$ With (from (a)(ii)) $\mathrm{f}''(0) = -6$ So 3rd term is $(-6)\frac{t^2}{2!} = -3t^2$	B1	**AG** Convincingly shown
				[1]	

Question			Answer	Marks	Guidance
7	(b)	(iii)	$\Rightarrow 5\cosh t + 5(t+1)\sinh t - 0.5\dfrac{\mathrm{d}x}{\mathrm{d}t} + \dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \dfrac{1}{2}\dfrac{\mathrm{d}^3 x}{\mathrm{d}t^3}$	M1 A1	For $\dfrac{\mathrm{d}}{\mathrm{d}t}(\cosh t) = \sinh t$ and product rule attempted Fully correct differentiation
			When $t = 0$, $5 + 0 + 2.5 - 6 = \dfrac{1}{2}\mathrm{f}'''(0)$ $\Rightarrow \mathrm{f}'''(0) = 3$	M1	Substitute $t = 0$
			$\Rightarrow$ 4th term $= \mathrm{f}'''(0)\dfrac{t^3}{3!} = \dfrac{1}{2}t^3$	A1	**AG**. Allow embedded answer.
				[4]	
	(c)	(i)	$x = 6 - 5t - 3t^2 + \dfrac{1}{2}t^3$ $\Rightarrow$ When $t = 0.25$, $x = 6 - 1.25 - 0.1875 - 0.0078 \approx 4.570...$ $\Rightarrow$ Distance travelled $6 - 4.570 \approx 1.430...$ So suitable as value close	B1	Substitutes $t = 0.25$ to obtain an approximation and correct conclusion
				[1]	
		(ii)	e.g. more terms may be required Higher terms may be large The candidate calculates the term in t^4 ($11t^4/12$) and indicates that this term is large for values of $t > 1$.	B1	Allow any correct explanation that explains/implies that for $t > 1$ some of the higher power terms are large and so non-negligible. f(10) = 156 is too large is **not** enough
				[1]	

Question			Answer	Marks	Guidance
8	(a)		$\overrightarrow{PQ}=\begin{pmatrix}2\\-2\\-2\end{pmatrix}, \overrightarrow{PR}=\begin{pmatrix}1\\1\\-3\end{pmatrix}$	**B1**	Both correct or other way round
			$\cos\theta=\dfrac{\begin{pmatrix}2\\-2\\-2\end{pmatrix}.\begin{pmatrix}1\\1\\-3\end{pmatrix}}{\sqrt{(2)^2+(-2)^2+(-2)^2}\times\sqrt{(1)^2+(1)^2+(-3)^2}}$ $=\dfrac{6}{\sqrt{12}\sqrt{11}}$	**M1**	Correct use of scalar product including correct method for magnitudes and dot product
			$\Rightarrow \sin\theta=\left(\sqrt{1-\left(\dfrac{6}{\sqrt{12}\sqrt{11}}\right)^2}\right)=\sqrt{\dfrac{8}{11}}=2\sqrt{\dfrac{22}{11^2}}=\dfrac{2}{11}\sqrt{22}$	**A1**	**AG** All working must be using exact forms
			Alternative for M1 A1 $\sin\theta=\dfrac{\left\lvert\begin{pmatrix}2\\-2\\-2\end{pmatrix}\times\begin{pmatrix}1\\1\\-3\end{pmatrix}\right\rvert}{\sqrt{(2)^2+(-2)^2+(-2)^2}\times\sqrt{(1)^2+(1)^2+(-3)^2}}$	**M1**	Correct use of vector product to find $\lvert\sin\theta\rvert$ including correct method for magnitudes, and correct method for $\begin{pmatrix}2\\-2\\-2\end{pmatrix}\times\begin{pmatrix}1\\1\\-3\end{pmatrix}$ soi. Condone $\sin\theta$ instead of $\lvert\sin\theta\rvert$
			$=\dfrac{4\left\lvert\begin{pmatrix}2\\1\\1\end{pmatrix}\right\rvert}{\sqrt{12}\sqrt{11}}=\dfrac{4\sqrt{6}}{\sqrt{132}}=\dfrac{4}{\sqrt{22}}=\dfrac{2}{11}\sqrt{22}$	**A1**	**AG**.
				[3]	

Question			Answer	Marks	Guidance
8	(a)		Alternative method: $\lvert QR\rvert=\sqrt{11},\ \lvert RP\rvert=\sqrt{11},\lvert PQ\rvert=\sqrt{12}$	B1	Sides of triangle
			Cosine rule $\Rightarrow \cos\theta=\dfrac{12+11-11}{2\times\sqrt{11}\times\sqrt{12}}=\sqrt{\dfrac{3}{11}}$	M1	Cosine rule
			$\Rightarrow \sin\theta=\sqrt{1-\dfrac{3}{11}}=\sqrt{\dfrac{8}{11}}=\dfrac{2}{11}\sqrt{22}$	A1	
			Or: $\lvert QR\rvert=\sqrt{11},\ \lvert RP\rvert=\sqrt{11},\lvert PQ\rvert=\sqrt{12}$	B1	Sides of triangle
			Drop perpendicular from R to QP at M $RM=\sqrt{11-\left(\dfrac{1}{2}\sqrt{12}\right)^2}=\sqrt{8}$	M1	Recognises an isosceles triangle so uses a median line
			$\Rightarrow \sin\theta=\sqrt{\dfrac{8}{11}}=\dfrac{2}{11}\sqrt{22}$	A1	
				[3]	

Question			Answer	Marks	Guidance
8	(b)		$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{pmatrix}2\\-2\\-2\end{pmatrix}\times\begin{pmatrix}1\\1\\-3\end{pmatrix}=4\begin{pmatrix}2\\1\\1\end{pmatrix}$	B1	BC or any other relevant vector product. Can be awarded if seen in (a)
			$\Rightarrow 2x+y+z=d$	M1	Attempt to use **r.n = *d*** to form linear equation. **ft** *their* vector product.
			Sub for a point $\Rightarrow d=5 \quad \Rightarrow 2x+y+z=5$	A1	Substitutes coordinates of P, Q or R to find d. (multiples accepted)
				[3]	
	(c)		$D=\dfrac{\left\|\begin{pmatrix}5\\3\\-1\end{pmatrix}.\begin{pmatrix}2\\1\\1\end{pmatrix}-5\right\|}{\left\|\begin{pmatrix}2\\1\\1\end{pmatrix}\right\|}=\dfrac{7}{\sqrt{6}}$	M1	Uses the formula given in formula book, or any other complete method for the shortest distance, **ft** *their* $\overrightarrow{PQ}\times\overrightarrow{PR}$.
				A1	Correct shortest distance.
			$V=\frac{1}{3}\times\left(\frac{1}{2}\left\|\overrightarrow{PQ}\right\|\left\|\overrightarrow{PR}\right\|\sin\theta\right)\times D$	M1	Uses $\frac{1}{2}\left\|\overrightarrow{PQ}\right\|\left\|\overrightarrow{PR}\right\|\sin\theta$oe, multiplied by their D/3, or $\frac{1}{2}\left\|\begin{pmatrix}2\\-2\\-2\end{pmatrix}\times\begin{pmatrix}1\\1\\-3\end{pmatrix}\right\|$ multiplied by their D/3, but must indicate that $\frac{1}{2}\left\|\begin{pmatrix}2\\-2\\-2\end{pmatrix}\times\begin{pmatrix}1\\1\\-3\end{pmatrix}\right\|$ is the area of the base.
			$=\frac{1}{3}\times\frac{1}{2}\times 2\sqrt{3}\times\sqrt{11}\times\frac{2}{11}\sqrt{22}\times\frac{7}{\sqrt{6}}$ $=\frac{14}{3}$	A1	**AG**.
				[4]	

Question			Answer	Marks	Guidance
8	(d)		$\begin{pmatrix}\cos\phi & 0 & \sin\phi\\ 0 & 1 & 0\\ -\sin\phi & 0 & \cos\phi\end{pmatrix}\begin{pmatrix}5\\3\\-1\end{pmatrix}=\begin{pmatrix}2\sqrt{2}\\ \ldots\\ \ldots\end{pmatrix}$	M1	For rotation matrix multiplied by $\overrightarrow{OR}$ or $\overrightarrow{OS}$.
			$\Rightarrow 5\cos\phi-\sin\phi=2\sqrt{2}$ $\Rightarrow \sin^2\phi=\left(5\cos\phi-2\sqrt{2}\right)^2$	M1	For a correct step to form quadratic equation in $\sin\phi$ or $\cos\phi$ only. For reference: $26\sin^2\phi+4\sqrt{2}\sin\phi-17=0$
			$\Rightarrow 1-\cos^2\phi=25\cos^2\phi-20\sqrt{2}\cos\phi+8$ $\Rightarrow 26\cos^2\phi-20\sqrt{2}\cos\phi+7=0$ $\Rightarrow \cos\phi=\dfrac{\sqrt{2}}{2},\ \dfrac{7\sqrt{2}}{26}$	A1	Solves quadratic equation in $\sin\phi$ or $\cos\phi$. (Exact answers required)
			$\sin\phi=5\cos\phi-2\sqrt{2}=\dfrac{\sqrt{2}}{2},\ -\dfrac{17\sqrt{2}}{26}$ $\begin{pmatrix}\cos\phi & 0 & \sin\phi\\ 0 & 1 & 0\\ -\sin\phi & 0 & \cos\phi\end{pmatrix}\begin{pmatrix}1\\3\\0\end{pmatrix}=\begin{pmatrix}\cos\phi\\ 3\\ -\sin\phi\end{pmatrix}$	M1	Uses their $\sin\phi$ or $\cos\phi$ with $5\cos\phi-\sin\phi=2\sqrt{2}$to find $\cos\phi$ or $\sin\phi$ respectively, (even if only one root) or fromsin $\phi=\sqrt{1-\cos^2\phi}$ or $\cos\phi=\sqrt{1-\sin^2\phi}$ (condone inclusion of $\pm$), or repeats previous method and multiplies out the matrices.
			$\Rightarrow \begin{pmatrix}\frac{\sqrt{2}}{2}\\ 3\\ -\frac{\sqrt{2}}{2}\end{pmatrix},\ \begin{pmatrix}\frac{7\sqrt{2}}{26}\\ 3\\ \frac{17\sqrt{2}}{26}\end{pmatrix}$ i.e. $R'=\left(\dfrac{\sqrt{2}}{2},3,-\dfrac{\sqrt{2}}{2}\right)$ or $\left(\dfrac{7\sqrt{2}}{26},3,\dfrac{17\sqrt{2}}{26}\right)$	A1	For both, and no others. Accept given as $\overrightarrow{OR'}$. If M0M0A0M0A0, SCB1 for any $R'$with y-coordinate = 3, and no other y-coordinates.
				[5]	

Question			Answer	Marks	Guidance
8	(d)		**Alternative method for first 3 marks:** $5\cos\phi - \sin\phi = 2\sqrt{2}$	M1	For rotation matrix multiplied by $\overrightarrow{OR}$ or $\overrightarrow{OS}$.
			$\Rightarrow 5\cos\phi - \sin\phi = \sqrt{26}\cos(\phi+\alpha) = 2\sqrt{2}$ where $\tan\alpha = \frac{1}{5} \Rightarrow \sin\alpha \frac{1}{\sqrt{26}}, \cos\alpha = \frac{5}{\sqrt{26}}$ $\sqrt{26}\sin(\phi+\alpha) = \pm\sqrt{26-\left(2\sqrt{2}\right)^2} = \pm 3\sqrt{2}$	M1	Expressing in the form $R\cos(\phi+\alpha)$ or $R\sin(\phi+\alpha)$ oe. Note that $5\cos\phi - \sin\phi$ $= \sqrt{26}\sin\left(\phi + \arctan\left(-\frac{1}{5}\right) + \pi\right)$
			$\cos\phi = \cos\left((\phi+\alpha)-\alpha\right) = \cos(\phi+\alpha)\cos\alpha + \sin(\phi+\alpha)\sin\alpha$ $= \frac{2\sqrt{2}}{\sqrt{26}} \times \frac{5}{\sqrt{26}} \pm \frac{3\sqrt{2}}{\sqrt{26}} \times \frac{1}{\sqrt{26}}$ $\Rightarrow \cos\phi = \frac{\sqrt{2}}{2},\ \frac{7\sqrt{2}}{26}$	A1	Solves for $\sin\phi$ or $\cos\phi$. For reference, $\frac{\sqrt{2}}{2} \approx 0.707, -\frac{17\sqrt{2}}{26} \approx -0.925$ and $\frac{7\sqrt{2}}{26} \approx 0.381$.
				[3]	

Question			Answer	Marks	Guidance
9	(a)		**DR** $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$ $\Rightarrow \left(e^{i\theta} - e^{-i\theta}\right)^4 = 16\sin^4\theta$ $\Rightarrow \left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$ $\Rightarrow \left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right) = \left(e^{4i\theta} + e^{-4i\theta}\right) - 4\left(e^{2i\theta} + e^{-2i\theta}\right) + 6$ $\Rightarrow 2\cos 4\theta - 8\cos 2\theta + 6 = 16\sin^4\theta$ $\Rightarrow \sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$ i.e. $A = \frac{1}{8}, B = -\frac{1}{2}, C = \frac{3}{8}$	**B1** **M1** **M1** **M1** **A1**	Or $e^{i\theta} + e^{-i\theta} = 2\cos\theta$ May use z without definition **oe**, eg. $(2i\sin\theta)^4 = 16\sin^4\theta = \left(e^{i\theta} - e^{-i\theta}\right)^4$. Award this mark for $\sin\theta$ to the power of four, and for $(2i)^4 = 16$. Note that 16 may appear later. Expanding $(e^{i\theta} - e^{-i\theta})^4$ with correct coefficients. Grouping terms and using $e^{i\theta} + e^{-i\theta} = 2\cos\theta$. cao, from fully correct reasoning. Allow A, B, C seen in the expression only.
				[5]	

Question			Answer	Marks	Guidance
9	(b)		**DR**		
			Let $u = x^{\frac{1}{5}} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{5}x^{-\frac{4}{5}}$	**B1**	Sight of $\frac{\mathrm{d}\left(\sin^{-1} u\right)}{\mathrm{d}u} = \frac{1}{\sqrt{1-u^2}}$
			Let $v = \sin^{-1} u \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}u} = \frac{1}{\sqrt{1-u^2}} = \frac{1}{\sqrt{1-x^{\left\langle\frac{2}{5}\right\rangle}}}$	**M1**	Uses chain rule
			$\Rightarrow \mathrm{f} = \sin 4v - 8\sin 2v + 12v \Rightarrow \frac{\mathrm{df}}{\mathrm{d}v} = 4\cos 4v - 16\cos 2v + 12$		
			$\frac{\mathrm{df}}{\mathrm{d}x} = \frac{\mathrm{df}}{\mathrm{d}v}.\frac{\mathrm{d}v}{\mathrm{d}u}.\frac{\mathrm{d}u}{\mathrm{d}x} = \left(4\cos 4v - 16\cos 2v + 12\right)\frac{1}{\sqrt{1-x^{\left\langle\frac{2}{5}\right\rangle}}} \times \frac{1}{5}x^{-\frac{4}{5}}$	**A1**	Correct derivative, f’
			$\frac{\mathrm{df}}{\mathrm{d}v} = 32\left(\frac{1}{8}\cos 4v - \frac{1}{2}\cos 2v + \frac{3}{8}\right) = 32\left(\sin v\right)^4 = 32u^4 = 32x^{\frac{4}{5}}$	**M1**	Uses result from (a)
				M1	Uses $\sin^4\left(\sin^{-1}\left(x^{\frac{1}{5}}\right)\right) = x^{\frac{4}{5}}$
			Then $\frac{\mathrm{df}}{\mathrm{d}x} = \frac{\mathrm{df}}{\mathrm{d}v}.\frac{\mathrm{d}v}{\mathrm{d}u}.\frac{\mathrm{d}u}{\mathrm{d}x} = 32x^{\frac{4}{5}} \times \frac{1}{\sqrt{1-x^{\frac{2}{5}}}} \times \frac{1}{5}x^{-\frac{4}{5}} = \frac{32}{5\sqrt{1-x^{\frac{2}{5}}}}$	**A1**	**AG** Clearly shown
				[6]	
9	(c)		$R = \lim_{k\to 1}\int_0^k \frac{1}{\sqrt{1-x^{\frac{2}{5}}}}\mathrm{d}x = \frac{5}{32}\lim_{k\to 1}\left[\mathrm{f}(x)\right]_0^k = \frac{5}{32}\lim_{k\to 1}\left(\mathrm{f}(k) - \mathrm{f}(0)\right)$	**M1**	For use of part (b) and an upper limit of $k < 1$ Integral must be found in terms of k (which could be 1)
			$\mathrm{f}(0) = 0$		
			$\mathrm{f}(k) = \sin\left(4\sin^{-1}\left(k^{\frac{1}{5}}\right)\right) - 8\sin\left(2\sin^{-1}\left(k^{\frac{1}{5}}\right)\right) + 12\sin^{-1}\left(k^{\frac{1}{5}}\right)$	**M1**	For correct use of limits
			As $k \to 1$ $\sin^{-1}\left(k^{\frac{1}{5}}\right) \to \sin^{-1}(1) = \frac{\pi}{2}$		
			$\Rightarrow \lim_{k\to 1}\left(\mathrm{f}(k)\right) = \sin\left(2\pi\right) - 8\sin\pi + 6\pi = 6\pi$		
			$\Rightarrow R = \frac{5}{32} \times 6\pi = \frac{15\pi}{16}$	**A1**	
				[3]	

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